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# Economic production quantity model with defective items, imperfect rework process, and lost sales

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**A B S T R A C T**

This study proposes an economic production quantity (EPQ) model that comprehensively addresses scrap items, imperfect quality items, rework processes, and shortages. The model incorporates various types of defective items, including scrap, imperfect quality, and reworkable items, and implements immediate rework processes upon the completion of regular production. Shortages are treated as lost sales, enhancing the accuracy of inventory cost estimations. Numerical experiments demonstrate the optimality of production lot sizes and underscore the impact of production and demand rate adjustments on overall inventory costs. Sensitivity analysis further determines the effect of imperfect quality items on inventory costs. This EPQ model offers a comprehensive approach to efficient and effective finished product inventory management by integrating considerations for scrap items, imperfect quality items, and rework processes. Additionally, a manufacturing company case is presented to illustrate the practical application of the proposed model.

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## 1. INTRODUCTION

### 1.1 Background

The economic order quantity (EOQ) model has long been fundamental in inventory management. The traditional EOQ model is built upon assumptions including constant annual demand, immediate order fulfilment, absence of quantity discounts, fixed setup costs, linear holding cost relationships, and perfect item quality [1]. While these assumptions simplify determining optimal order or production quantities, real-world situations often diverge, particularly in item deterioration. Deterioration typically refers to items undergoing defectives, being discarded, becoming outdated, undergoing alterations, or being perishable [2]. As a result, researchers have explored alternative models like economic production quantity (EPQ), which consider factors associated with imperfect production processes and defective items. Hsu and Hsu [1] introduced a model where defective items are treated as a random variable, emphasizing the practice of discarding identified defective items. However, maximizing manufacturing resources requires a shift towards strategies that enable the repair of defective items to meet demand or reintegrating them into the production process as raw materials. Pasandideh *et al.* [2] further categorized defective items based on the severity of defects.

Recent research has emphasized extending traditional EPQ models to incorporate imperfect production processes and rework operations. Taleizadeh *et al.* [3] extended the EPQ model to accommodate imperfect rework processes, highlighting the evolving landscape of inventory management practices. Mokhtari [4] integrated EPQ and EOQ models to optimize production and replenishment strategies. Similarly, Moussawi-Haidar *et al*. [5] proposed an imperfect EPQ model that included a screening mechanism for defective goods during production.

Moreover, various studies have proposed EPQ models tailored to specific scenarios, including those considering damaged or imperfect goods and rework processes [6]. Nobil and Taleizadeh [7] developed a model for the joint determination of cycle time in a multi-item EPQ setting, accounting for random defect rates and interruptions in the production process and scrap and rework. Kalantari and Taleizadeh [8] developed a model to manage time-varying damage rates, aiming to determine the optimal replenishment lot size that minimizes the long-term average cost function. Another EPQ model, focusing on replenishment optimization while satisfying stochastic constraints and determining the optimal number and volume of lots, was introduced by Gharaei *et al*. [9]. Öztürk [10] devised an economic production quantity model for imperfect production systems, incorporating repairs for nonconforming products and managing shortages with a last-in/first-out (LIFO) policy.

A model for optimizing the number of inspections frequency, intervals, and economic production quantity under the assumption that the production process is in a controlled state or an out-of-control state has been proposed by Lin *et al*. [11]. Viji and Karthikeyan [12] introduced an EPQ model to minimize the total cost of the production cycle, considering different production levels and a two-parameter Weibull distribution for defective rates, along with backorders to fulfil inventory requirements. Furthermore, Ganesan and Uthayakumar [13] established two production inventory models that included preparation runs of production, hybrid maintenance schedules, expected shortages during maintenance periods, and the rework of imperfect products. Additionally, several studies have focused on EPQ models considering imperfect quality items, where defective items produced during production must undergo rework to determine the optimal production lot size, accounting for additional costs and potential backorders [10], [14].

According to Silver *et al.* [15], handling customer orders during temporary out-of-stock situations is critical to inventory control, presenting two primary scenarios: total reordering and complete lost sales. However, it is important to note that some companies encounter lost sales. In such instances, when a product is unavailable, customer demand for that item is forfeited, and competitors may also meet it. The costs related to lost sales can vary, encompassing direct profit losses on the affected sales as well as more abstract losses linked to missed opportunities. For example, such missed opportunities may discourage customers from future visits to the store for other items, exerting a lasting impact on customer retention and overall business performance [16].

Despite recent progress, the literature lacks a comprehensive EPQ model integrating rework processes, scrap, imperfect quality items, and shortages. Thus, this study aims to enhance the efficiency and effectiveness of managing finished product inventories by proposing an EPQ model that considers scrap items, imperfect quality items, rework processes, and shortages. The model is adapted from the framework introduced by Öztürk [10]; the objective is to optimize production lot sizes while accounting for the influence of rework processes, scrap, and imperfect quality items produced during both production and rework phases, excluding consideration of backorders. Additionally, the study draws upon the research conducted by Hadley and Whitin [17] to establish the shortage cost incorporated into model Q, treating inventory shortages as lost sales to address them effectively.

The main contributions of this research are described as follows:

1) Addressing the current economic production quantity (EPQ) model gap by integrating rework processes for defective items and acknowledging lost sales due to shortages. Shortages are treated as lost sales, reflecting consumer unwillingness to wait for product restocking.
2) Leveraging the EPQ framework within inventory management models to comprehensively evaluate the total cost.
3) Offering an analytical solution to the optimization problem by extending the model and deriving the EPQ equation for production lot size. This extension is validated through numerical experimentation.

### 1.2 Research gap

The EOQ model was formulated by Harris in 1913. Subsequently, Taft, in 1918, extended and refined the model, renaming it the EPQ model [18].


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The EPQ model minimises total production costs by optimising inventory costs [19]. According to Ballou [20], the EPQ model relies on three fundamental parameters: demand, production setup cost, and inventory holding cost per unit. However, Kostić [21] argued that these parameters are insufficient to address real-world challenges adequately, as the traditional EPQ model does not consider realistic company conditions such as imperfect quality and defective items, imperfect rework process, and lost sales.

Wee *et al.* [22] enhanced the model's functionality by incorporating considerations for varying defective rates, acknowledging the inherent variability in production processes. Meanwhile, *Jaber et al.* [23] introduced an assumption where the proportion of defective items per lot diminishes over time, following a learning curve. It implies that with repeated production cycles, there is an expected improvement in product quality due to the learning and experience gained. Mukhopadhyay and Goswami [24] delved into the complexities of inventory management by investigating an economic production quantity model designed to accommodate three distinct types of imperfect items. Their research also addressed reworking defective items to salvage their value, thus minimizing waste and maximizing resource utilization.

Schrady [25] was a pioneer in the earliest studies that focused on rework and remanufacturing processes. Subsequently, studies related to rework processes have garnered significant attention from researchers [4]–[7], [26]. Rework processes involve repairing or modifying defective items to restore their quality, thereby minimizing waste and maximizing resource utilization in manufacturing operations [27]. Moreover, Ebrahimi *et al.* [28] developed a model to integrate sustainability principles, uncertain multi-objective optimization, lost sales, and full back-order considerations into economic production quantity modeling.

According to the analysis provided in Table 1 from previous studies, the EPQ model has been extensively employed to tackle inventory management challenges. Previous researchers primarily utilized EPQ models that accounted for shortages as a crucial element of their methodology. Additionally, certain studies incorporated considerations for defective items necessitating rework. This study proposes an EPQ model specifically tailored to inventory scenarios where shortages are regarded as complete lost sales. Notably, the parameters employed in this study align with a study by Öztürk [10], which predominantly addressed shortage backorders. Therefore, our study aims to contribute to the literature by addressing shortages as wholly lost sales, enhancing the understanding of inventory management dynamics in this context.

The structure of this paper is as follows: Section 2 outlines the research methodology, Section 3 presents the results and discussion, and finally, Section 4 offers the conclusions.

**Table 1.** Literature review on EPQ

| Author | Method | *Shortage* | | | *Based Classification* | | *Parameter* | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *Backorder* | | *Lost Sale* | *Single Item* | *Multi Item* | *Defective Items* | *Scrap Items* | *Rework Process* |
| | | *Fully* | *Partially* | | | | | | |
| Najafi *et al.* [14] | EPQ | | √ | | | √ | √ | √ | √ |
| Viji and Karthikeyan [12] | EPQ | √ | | | √ | | | | |
| Al-Salamah [29] | EPQ | √ | | | √ | | √ | | √ |
| Sanjai and Periyasamy [30] | EPQ | | √ | | √ | | √ | | √ |
| Ganesan and Uthayakumar [13] | EPQ | | √ | | √ | | √ | | √ |
| Guha and Bose [31] | EPQ | | | | √ | | √ | | √ |
| Malik and Sarkar [32] | EPQ | | √ | | | √ | | | |
| Öztürk [10] | EPQ | √ | | | √ | | √ | √ | √ |
| Castellano and Glock [33] | EPQ | √ | | | √ | | √ | | √ |
| Karagül and Eroğlu [34] | EPQ | √ | | | √ | | √ | | √ |
| Priyan *et al.* [35] | EPQ | √ | | | √ | | √ | | √ |
| This research | EPQ | | | √ | √ | | √ | √ | √ |

## 2. RESEARCH METHODS

### 2.1. Assumptions and notation

This model considers a production process that produces non-defective items at a constant rate *P*, where *P* is greater than *D*. The procedure also generates defective items at a rate *d* and a defect rate of *q* percent. Every item produced are subjected to screening, which costs *c* per unit. Defective items are classified into three categories: scrap, imperfect quality items, and reworkable items. The imperfect items are randomly sorted according to proportion θ and removed from inventory during each production and rework cycle. Imperfect quality items (expressed by a random proportion γ) are sold at a lower price at the end of the production phase. Furthermore, rework begins immediately after the regular production period ends, addressing additional defective items at a rework rate, where $P_1$ is greater than D.

Imperfect rework involves a random proportion $\theta_1$ of reworked items, with a portion having imperfect quality items denoted by $\gamma_1$. The rate of scrap item production during rework ($d = Pq$), the rate of defective item production during rework ($d_1 = P_1\theta_1$), and the imperfect quality items during rework ($d_2 = P_1\gamma_1$).

The model accounts for lost sales upon inventory exhaustion or within a specified lead time *N*. Assumptions include a single product, constant and continuous demand rate, constant and known production and rework rates (P > D, $P_1$ > D) simultaneous production and rework within a cycle and allowance for complete lost sales due to shortages.

The notation used for model formulation is:

| | | |
|---|---|---|
| $P$ | : | Production rate |
| $P_1$ | : | Rework rate |
| $Q$ | : | Production lot size |
| $D$ | : | Demand rate |
| $d$ | : | Production rate of defective items |
| $d_1$ | : | Production rate of scrap items during rework |
| $d_2$ | : | Production rate of imperfect items during rework |
| $d^{\circ}$ | : | Sum of the production rate of scrap items and imperfect quality items during rework, $d^{\circ} = d_1 + d_2$ |
| $t_1$ | : | Time period when inventory accumulates |
| $t_2$ | : | Time period for rework |
| $t_3$ | : | Time period to deplete inventory |
| $T$ | : | Cycle length |
| $q$ | : | Proportion of defective items produced |
| $\theta$ | : | Proportion of scrap items in defective items |
| $\gamma$ | : | Proportion of imperfect quality items in defective items |
| $\beta$ | : | Proportion of reworkable items during rework |
| $\theta_1$ | : | Proportion of scrap items produced during rework |
| $\gamma_1$ | : | Proportion of imperfect quality items produced during rework |
| $E$ | : | Expected value operator |
| $L$ | : | Inventory shortage rate |
| $K$ | : | Setup cost for each production run |
| $H_1$ | : | Maximum rate of perfect item inventory at the end of regular production run |
| $H$ | : | Maximum rate of perfect item inventory at the end of the rework process |
| $c$ | : | Production cost |
| $c_s$ | : | Scrap items disposal cost |
| $h$ | : | Holding cost |
| $h_1$ | : | Holding cost of reworkable items |
| $c_R$ | : | Rework cost of reworkable items |
| $c_u$ | : | Lost sales cost |
| $S\sqrt{L}$ | : | Standard deviation over lead time |
| $S$ | : | Standard deviation |
| $L$ | : | Lead time |
| $N$ | : | Expected inventory shortage during lead time |
| $z\alpha$ | : | Normal deviation |
| $\alpha$ | : | Probability of lost sales |
| $f(z\alpha)$ | : | Ordinate |
| $\Psi(z\alpha)$ | : | Partial expectation |

### 2.2. Model formulation

The EPQ model utilized in this study is a modified version of the EPQ model developed by Öztürk [10]. The model formulation begins with the delineation of periods within the production cycle (Fig. 1). Period $t_1$ denotes the duration allocated for production, followed by $t_2$, which signifies the period allocated for the rework process. Production ceases at the time $t_3$, marking the end of the entire cycle. After regular production (at the time $t_1$), the inventory rate $H_1$ is initiated, representing the inventory accumulation during this phase. Subsequently, as the rework process concludes ($t_2$), the maximum

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inventory rate $H$ is attained, signifying the peak inventory level at the end of the rework process.

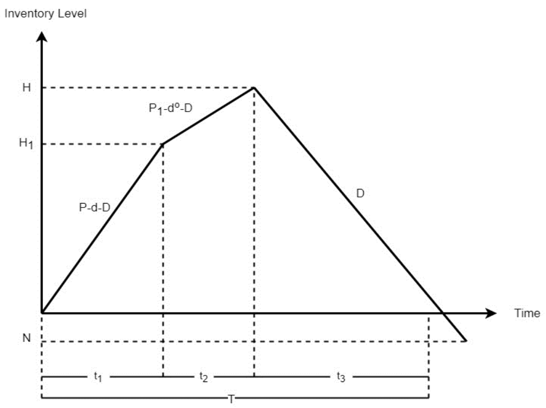


**Fig. 1**. EPQ model graph

$$t_1 = \frac{H_1}{P - d - D} \quad (1)$$

$$t_2 = \frac{H - H_1}{P_1 - d^{\circ} - D} \quad (2)$$

$$t_3 = \frac{H}{D} \quad (3)$$

$$t_2 = \frac{\beta d Q}{P_1 P} = \frac{\beta q Q}{P_1} \quad (4)$$

$$H_1 = (P - d - D)t_1 \quad (5)$$

$$H = H_1 + (P_1 - d^{\circ} - D)t_2 \quad (6)$$

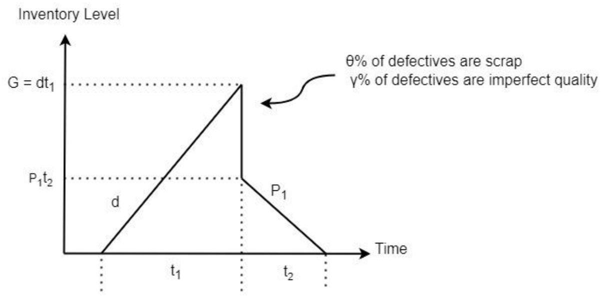


**Fig. 2.** EPQ graph model inventory rate of defective items

Fig. 2 shows defective items produced during the regular production period ($t_1$) are computed as in equation (7).

$$G = dt_1 = qQ \quad (7)$$

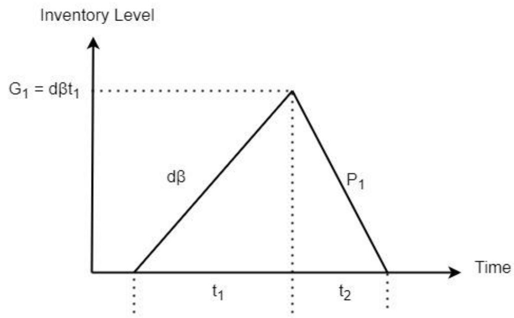


**Fig. 3.** EPQ graph of reworkable items inventory rate model

Fig. 3 shows that the defective items are classified as reworkable items (β), scrap items (θ), or imperfect quality (γ). Consequently, the quantities of reworkable items, imperfect quality items, and scrap items generated within the regular production process can be represented as the following equations (8), (9), and (10), respectively:

$$G_1 = d\beta t_1 = P_1 t_4 = \beta q Q \quad (8)$$

$$G_2 = d\gamma t_1 = \gamma q Q \quad (9)$$

$$G_4 = d\theta t_1 = \theta q Q \quad (10)$$

Fig. 4 and Fig. 5 illustrate that, during the rework process, the scrap item rate and defective item production rate during the rework process, can be expressed as in equations (11) and (12), respectively. Consequently, the total defective items include scrap items and defective items produced during this process.

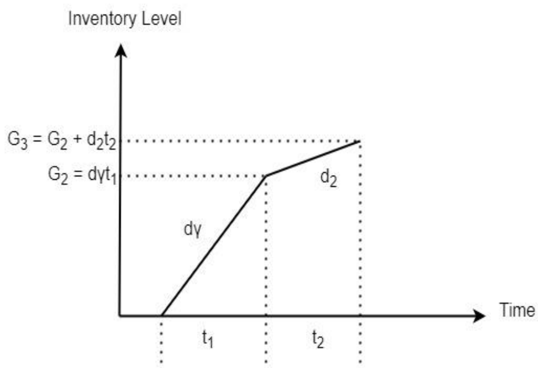


**Fig. 4.** EPQ graph of imperfect inventory rate model

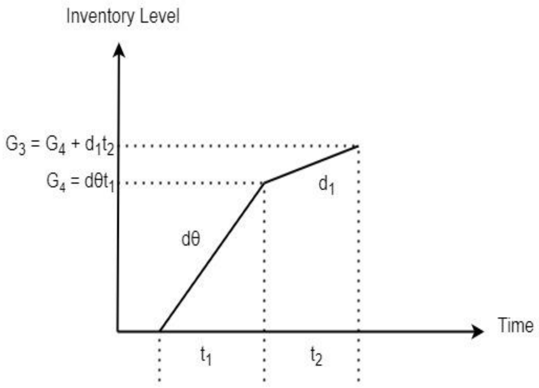


**Fig. 5.** EPQ graph of scrap items inventory rate model

The number of items produced can be determined using equations (13) and (14). Additionally, $d^{\circ}$ represents the sum of the production rates of scrap items and defective items produced during rework, as described in equation (15). Consequently, the inventory rates $H_1$ and H can be obtained as follows:

$$d_1 = P_1\theta_1 \quad (11)$$

$$d_2 = P_1\gamma_1 \quad (12)$$

$$G_3 = G_2 + d_2t_2 = (\gamma + \gamma_1\beta)qQ \quad (13)$$

$$G_5 = G_4 + d_1t_2 = (\theta + \theta_1\beta)qQ \quad (14)$$

$$d^{\circ} = d_1 + d_2 = P_1(\theta_1 + \gamma_1) \quad (15)$$

$$H_1 = (P - d - D)t_1 = A_1Q \quad (16)$$

$$H = H_1 + (P_1 - d^{\circ} - D)\frac{\beta qQ}{P_1} = A_1Q + A_2\beta qQ = (A_1 + A_2\beta q)Q \quad (17)$$

$$P - d - D = P\left(1 - q - \frac{D}{P}\right) = PA_1 \quad (18)$$

$$P_1 - d^{\circ} - D = P_1\left(1 - \theta_1 - \gamma_1 - \frac{D}{P_1}\right) = P_1A_2 \quad (19)$$

Thus, the cycle length (T) is obtained as follows:

$$T = t_1 + t_2 + t_3 \quad (20)$$

With the addition of the lost sales cost based on the model by Hadley and Whitin [17], the formula for updating the total inventory cost per cycle TC(Q) is obtained as follows:

$$TC(Q) = K + cQ + c_s(\theta + \theta_1\beta)qQ + c_R\beta qQ + h\left(\frac{E_2Q}{2P} + \frac{(E_2Q^2 + (E_2 + A_2\beta E(q))Q^2)\beta E(q)}{2P_1} + \frac{(E_2 + A_2\beta E(q))^2Q^2}{2D} + \frac{P - P(1 - E(q))Q}{2P}\right) + \frac{h_1\left(P - P(1 - E(q))\right)\beta^2Q^2E(q)}{2PP_1} + C_lL \quad (21)$$

As the proportion of defective items constitutes a random variable, the production cycle length (T) similarly becomes a random variable. Subsequently, the expected cycle length, denoted as E(T), is determined according to Öztürk [10].

$$E(T) = \frac{[1 - E(q)(1 - \beta(1 - \theta_1 - \gamma_1))]Q}{D} = \frac{(1 - E_1)Q}{D} \quad (22)$$

Where:

$$E_1 = E(q)(1 - \beta(1 - \theta_1 - \gamma_1)) \quad (23)$$

$$E_2 = 1 - E(q) - \frac{D}{P} \quad (24)$$

$$E_3 = E\left(\frac{q}{1 - q - \frac{D}{P}}\right) \quad (25)$$

$$E_4 = E\left(\frac{1}{1 - q - \frac{D}{P}}\right) \quad (26)$$

$$E_5 = 1 - \theta_1 - \frac{D}{P_1} \quad (27)$$

The formula for the expected inventory shortage during lead time (N), derived from the model proposed by Hadley and Whitin [17], is as follows:

$$N = S\sqrt{L}(f(z\alpha) - z\alpha.\Psi(z\alpha)) \quad (28)$$

The total inventory cost in units of expected cycle length is obtained:


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$$E(TC(Q^*)) = \frac{E(TC(Q))}{E(T)} = \frac{KD}{(1-E_1)Q} + \frac{(c + E(q)[c_s(\theta + \theta_1\beta) + c_R\beta])D}{(1-E_1)} + \frac{1}{(1-E_1)}\left(h\left(\frac{E_2Q}{2P} + \frac{(E_2Q + (E_2 + A_2\beta E(q))Q)\beta E(q)}{2P_1} + \frac{(E_2 + A_2\beta E(q))^2 Q}{2D} + \frac{P - P(1-E(q))}{2P}\right)D\right) + \frac{h_1\left(P - P(1-E(q))\right)\beta^2 QE(q)D}{2PP_1(1-E_1)} + \frac{C_lLD}{(1-E_1)Q} \quad (29)$$

To minimize the expected total inventory cost $\left(E\left(TC(Q^*)\right)\right)$ where for all Q > 0. So, the first derivative $\left(E\left(TC(Q^*)\right)\right)$ is obtained against Q, as follows:

$$\frac{\partial E(TCU(Q))}{\partial Q} = -\frac{KD}{(1-E_1)Q^2} + \frac{h\left(\frac{E_2}{2P} + \frac{(A_2\beta E(q)+2E_2)\beta E(q)}{2P_1} + \frac{(A_2\beta E(q)+E_2)^2}{2D}\right)D}{(1-E_1)} + \frac{h_1\left(P - P(1-E(q))\right)\beta^2 E(q)D}{2PP_1(1-E_1)} - \frac{C_lLD}{(1-E_1)Q^2} = 0 \quad (30)$$

Thus, determining the optimum production lot size ($Q^*$) is obtained from the expected total inventory cost derived for Q, which equals 0. So that the following equation is obtained:

$$Q^* = \sqrt{\frac{2DPP_1(C_lL + K)}{h(DE_2 + P(A_2\beta E(q) + E_2)^2)P_1 + DPE(q)\beta\left(A_2\beta hE(q) + \beta h_1E(q) + 2E_2h\right)}} \quad (31)$$

Due to the imperfect production process, where the proportion of defective items (q), the rate of producing perfect items for each cycle is $P(1-q) = P - d$. It should be noted that to ensure the production process meets demand with perfect items during the production period ($t_1$), the production rate must always exceed the demand rate.

$$P - d > D \ or \ \left(1 - q - \frac{D}{P}\right) > 0 \quad (32)$$

Additionally, to prevent shortages during the rework process, the inventory quantity ($H_1$) at the beginning of the rework time ($t_2$) and the perfect item quantity, even after the rework period, should always be greater than the demand.

$$H_1 + (P_1 - d^{\circ})t_2 > Dt_2 \quad (33)$$

## 3. RESULTS AND DISCUSSION

In this study, a solution analysis using a sensitivity approach is conducted with an economic production quantity model.

### 3.1. Numerical data

Numerical data is presented to validate the application of the proposed model in determining the optimal production lot size. Data collection for this study was conducted through observations, interviews with company representatives, and extensive review of pertinent research literature. The parameter model in this experiment is listed in Table 2. The parameter values for q, θ, γ, β, $\theta_1$, and $\gamma_1$ have been adopted from the research conducted by Öztürk [10].

**Table 2.** Experiment parameters

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| D | 4,030 units/year | $h_1$ | Rp 21,400 /(unit/year) |
| P | 12,000 units/year | $c_R$ | Rp 63,687 /unit |
| $P_1$ | 5,000 units/year | $c_L$ | Rp 223,202 /unit |
| $S\sqrt{L}$ | 4 units/year | $E(q)$ | 0.05 |
| $z\alpha$ | 1.65 | $q$ | 0.05 |
| $f(z\alpha)$ | 0.1023 | $\theta$ | 0.1 |
| $\Psi(z\alpha)$ | 0,0206 | $\gamma$ | 0.2 |
| K | Rp 777,935 for each production | $\beta$ | 0.7 |
| c | Rp 231,253 /unit | $\theta_1$ | 0.1 |
| $c_s$ | Rp 25,012 /unit | $\gamma_1$ | 0.3 |
| h | Rp 32,792 /(unit/year) | | |

### 3.2. Finding optimal solution

The objective of this research is to determine the optimal production lot size while calculating the total inventory cost. Equation (19) is derived by modifying the Öztürk [10] model to acquire the $A_2$ value, which will then be applied in the computation of production lot size and the expected total inventory cost as presented in equations (31) and (29). The $A_2$ value obtained from equation (19) is -0.21, demonstrating a negative result consistent with the negative finding reported by Öztürk [10]. Using equation (28), the value of the expected inventory shortage during the lead time (N) equals one. Furthermore, the expected values $E_1$ and $E_2$ are determined from a uniform distribution taken from Öztürk [10], resulting in values of 0.03 and 0.61, respectively. Next, the expected total inventory cost $E(TC(Q^*))$ is calculated using equation (29), yielding a result of Rp 987,344,037. Using equation (31), the optimal production lot size $(Q^*)$ is found to be 639 units.

The production lot size greatly influences the total inventory cost, as shown in Fig. 6. The expected total inventory cost decreases as the size of the production lot increases. Furthermore, the total inventory cost rises after reaching the optimal production lot size of 639.

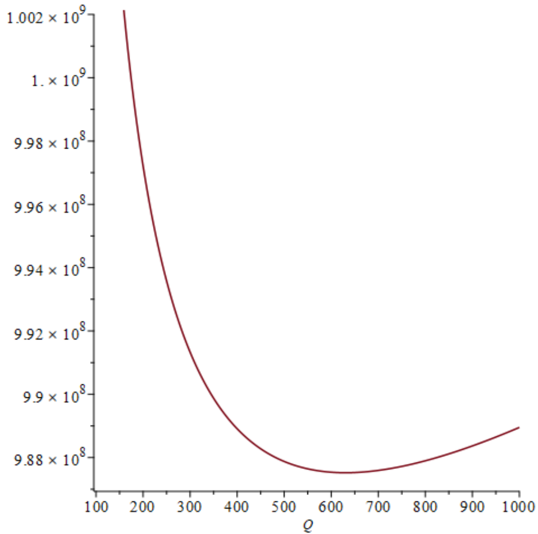


**Fig. 6.** Effect of changes in $Q^*$ on expected total inventory cost

### 3.3. Sensitivity analysis

Sensitivity analysis is performed to determine how the effect of the total inventory cost results in changes in the value of each parameter. The sensitivity analysis is performed on various parameters, including the production rate (P), demand rate (D), defective items $E(q)$, reworkable items $(\beta)$, scrap items produced during rework $(\theta_1)$ and imperfect quality items produced during rework $(\gamma_1)$ across five trials. This is done to assess how changes in these parameters affect the total inventory cost results.

As the production rate increases, the expected total inventory cost $E\big(TC(Q^*)\big)$ also increases, as demonstrated in Table 3. However, the resulting production lot size has decreased. For this reason, the company can adopt a better policy to increase production rates to minimize the expected total inventory cost. Similarly, the expected total inventory costs will increase as the demand rate of value rises. The results show that if the value of defective items $E(q)$ increases, the expected total inventory cost $E\big(TC(Q^*)\big)$ incurred also increases. For this reason, companies can take policies to reduce $E(q)$ as a solution

The economic production quantity (EPQ) calculation reveals that the inventory cost associated with reworkable items during the rework phase (β) decreases as this value increases. This indicates that the total inventory costs are influenced by processing defective items into reworkable ones. Conversely, the situation differs concerning the proportion of scrap items $(\theta_1)$, and imperfect quality items during rework $(\gamma_1)$, as the expected total inventory cost increases. Between these two parameters, the reduction of imperfect quality items during rework $(\gamma_1)$ has a more favourable impact on minimizing the expected total inventory cost

### 3.4. Managerial implications

The research offers several key insights companies should consider when planning and managing their production and inventory processes.

1. Companies need to implement effective strategies to manage defective products and imperfect rework processes to minimize scrap and optimize production lot sizes, cycle times, and total inventory costs. Furthermore, companies must incorporate variations in product demand into their production planning to ensure sufficient product availability. Failure to do so can lead to inefficiencies and increased production costs, impacting overall profitability.
2. Lost sales due to inventory shortages must be


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**Table 3.** Sensitivity analysis

| Parameter | Change | $Q^*$ | $E(TC(Q^*))$ |
|---|---|---|---|
| P | 10800 | 653 | 987,066,974 |
| | 11400 | 642 | 987,287,001 |
| | 12000 | 639 | 987,344,037 |
| | 12600 | 627 | 987,582,411 |
| | 13200 | 624 | 987,656,552 |
| D | 3626 | 585 | 889,438,649 |
| | 3828 | 612 | 938,403,106 |
| | 4030 | 639 | 987,344,037 |
| | 4232 | 658 | 1,036,439,566 |
| | 4434 | 686 | 1,085,340,496 |
| $E(q)$ | 0.03 | 628 | 972,132,388 |
| | 0.04 | 633 | 974,700,859 |
| | 0.05 | 639 | 987,344,037 |
| | 0.06 | 644 | 989,938,516 |
| | 0.07 | 650 | 1,002,871,622 |
| $\beta$ | 0.5 | 643 | 994,768,126 |
| | 0.6 | 641 | 985,928,366 |
| | 0.7 | 639 | 987,344,037 |
| | 0.8 | 637 | 988,759,796 |
| | 0.9 | 635 | 980,071,811 |
| $\theta_1$ | 0.1 | 639 | 987,344,037 |
| | 0.2 | 641 | 987,661,170 |
| | 0.3 | 643 | 998,269,846 |
| | 0.4 | 646 | 998,590,481 |
| | 0.5 | 648 | 998,911,219 |
| $\gamma_1$ | 0.1 | 634 | 977,361,582 |
| | 0.2 | 636 | 987,390,712 |
| | 0.3 | 639 | 987,344,037 |
| | 0.4 | 641 | 987,297,465 |
| | 0.5 | 643 | 997,534,858 |

Factored into the overall cost analysis, it includes not only the immediate revenue loss but also potential long-term implications on customer satisfaction and brand reputation.

3. The EPQ framework can help companies determine optimal production lot size cycle times and minimize total inventory costs. This approach should now consider complete shortages as lost sales rather than treating them solely as backorders.
4. An analytical approach integrating the EPQ equation to determine production lot sizes while considering lost sales occurrences aims to reduce instances of unmet demand and minimize inventory costs. However, ongoing research should address limitations such as random defect rates and fixed demand levels, exploring multi-item products and other relevant factors for a comprehensive model.

## 4. CONCLUSION

This study provides valuable insights into determining the optimal production lot size through the Economic Production Quantity (EPQ) model, enabling the formulation of more efficient production policies to address shortages and mitigate lost sales effectively. The model demonstrates its applicability and effectiveness in minimizing overall product costs by comprehensively considering various forms of imperfect products, such as scrap and reworkable imperfect quality. The managerial implications of these findings underscore the importance of strategic decision-making in inventory management, highlighting the significant impact of the optimal production lot size on total production costs. Future research endeavors could further enhance the practical applicability of the model by incorporating considerations for multi-item products and warehouse constraints, thus contributing to more robust and adaptable inventory management strategies.

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