

ANALYSIS OF RESIDUAL CHLORINE IN WATER PRODUCTION USING MEC CONTROL CHARTS WITH FAST INITIAL RESPONSE FEATURES

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Abstract

Problems: Residual chlorine is a crucial parameter in water production processes to ensure microbiological safety. However, conventional statistical process control charts often exhibit limited sensitivity to detecting small, early shifts, particularly during the initial monitoring (startup) phase. Excessive or insufficient residual chlorine levels may pose health risks and indicate instability in the production process, highlighting the need for more responsive monitoring tools.

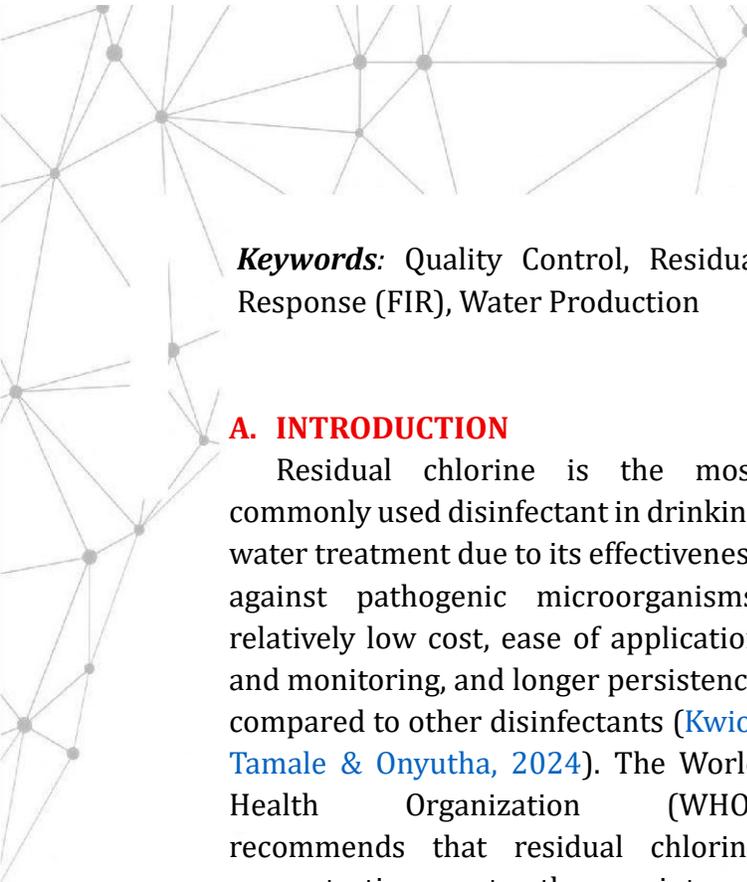
Purpose: This study aims to evaluate the performance of the Mixed EWMA–CUSUM (MEC) control chart integrated with Fast Initial Response (FIR) and Modified Fast Initial Response (MFIR) features in monitoring free residual chlorine levels in water production systems.

Methodology: This study employs a quantitative analytical approach using secondary data obtained from a real water production process. The MEC control chart combines the strengths of EWMA and CUSUM to improve sensitivity to small shifts, while incorporating FIR and MFIR features to enhance early detection during the startup phase. Various parameter combinations are examined to assess detection behavior and control limit characteristics.

Results/Findings: The results indicate that the MEC control chart with MFIR features provides earlier and more sensitive detection of potential process deviations compared to conventional approaches. In particular, the MFIR chart with parameters $f = 0.75$ and $\lambda_s = 0.75$ produces narrower control limits during the initial phase and identifies three out-of-control observations. These findings demonstrate that integrating MFIR into the MEC framework enhances early-stage monitoring performance and offers practical benefits for residual chlorine surveillance in water production systems.

Paper Type: quantitative analytical research

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Keywords: Quality Control, Residual Chlorine, MEC Control Chart, Fast Initial Response (FIR), Water Production

A. INTRODUCTION

Residual chlorine is the most commonly used disinfectant in drinking water treatment due to its effectiveness against pathogenic microorganisms, relatively low cost, ease of application and monitoring, and longer persistence compared to other disinfectants (Kwio-Tamale & Onyutha, 2024). The World Health Organization (WHO) recommends that residual chlorine concentrations at the point of consumption be maintained within the range of 0.2–5 mg/L to protect public health from secondary microbial contamination in treated water supplies (World Health Organization, 2017). However, maintaining residual chlorine within this recommended range is challenging due to process variability during water production. Insufficient chlorine levels may result in microbiological contamination, while excessive concentrations can cause adverse health effects, such as skin and eye irritation. Therefore, effective quality control mechanisms are essential to ensure process stability and consistent compliance with water quality standards.

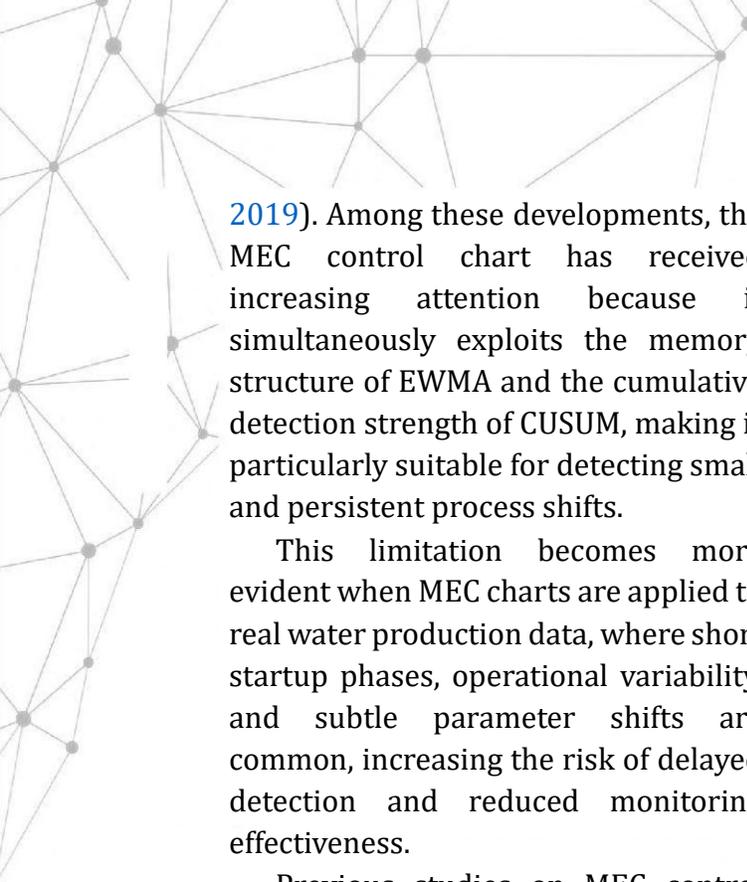
Statistical Process Control (SPC) techniques, particularly control charts, have been widely applied to monitor process performance and detect

deviations in water treatment systems. Conventional Shewhart control charts, however, are limited in their ability to detect small and gradual shifts because they rely primarily on current observations. To overcome this limitation, alternative charts such as the Cumulative Sum (CUSUM) chart by Page and the Exponentially Weighted Moving Average (EWMA) chart by Roberts were developed. Both charts are more effective at detecting small shifts in the process average than the Shewhart chart (Montgomery, 2009). CUSUM charts track cumulative deviations from the target value, while EWMA charts exponentially weight historical observations. Although the performance is comparable, CUSUM is easier to implement (Montgomery, 2009). These graphs are commonly used to detect changes in process quality and mean shifts in production systems. Subsequent research has further investigated EWMA graphs, including run-length distribution analysis using the integral equation approach (Crowder, 1987) and the Markov chain method (Lucas & Saccucci, 1990). Additional features, such as fast initial response (FIR), combined Shewhart-EWMA graphs, and special rules such as “two-in-a-row”, have been developed to improve the performance of EWMA.

Several modifications to the EWMA chart have also been proposed, such as the double EWMA (DEWMA) chart (Shamma & Shamma, 1992), which adjusts the control limits asymptotically to better detect small shifts. Adaptive EWMA (AEWMA) chart for detecting both small and large shifts simultaneously (Capizzi & Masarotto, 2003). Mixed EWMA-CUSUM (MEC) charts, which combine the design strengths of EWMA and CUSUM, making them more sensitive to small process shifts (Abbas et al., 2013). MEC charts integrate EWMA statistics into the CUSUM framework, thus achieving higher sensitivity in detecting small mean shifts (Osei-Aning et al., 2017). MEC charts have been applied in various studies, including for monitoring water production quality (Asmara et al., 2021). In addition, MEC is also used in flour production (Sari et al., 2024).

The MEC control chart has been widely extended to improve its ability to detect shifts in the process mean. Among these developments are expanded mixed control charts that incorporate a new median estimator, the ranked set sampling procedure, and the double ranked set sampling procedure (Mohamadkhani & Amiri, 2020). In addition, MEC control charts have been extended to accommodate non-normal data. In the presence of outliers, which are a primary source of non-normality, chart parameters may be overestimated, thereby reducing the

reliability of the monitoring process. To address this issue, robust median-based estimators, specifically the median and the modified one-step M-estimator (MOM), are employed to estimate and control location parameters within the MEC control chart framework under non-normal distributional conditions (Mohd Noor et al., 2023). In this paper, to further improve the sensitivity of MEC charts, a fast initial response (FIR) feature has been integrated. Lucas & Crosier initially introduced FIR to CUSUM charts to improve performance at startup or after an out-of-control signal (Lucas & Crosier, 2000). This feature was later extended to EWMA charts, which showed superior detection capabilities compared to standard EWMA charts (Rhoads et al., 1996). Steiner formalized the FIR adjustment by introducing the Basic FIR (BFIR), which tightens the control limits during startup and converges to the standard limits over time (Steiner, 1999). Subsequently, the BFIR feature developed a modified version, MFIR, to further improve sensitivity (Haq et al., 2014). The FIR feature has been widely applied in several previous studies, such as to monitor the coefficient of variation (Hu et al., 2022), the addition of FIR to the EWMA-t control chart (Haq et al., 2019), Mixed EWMA Dual-CUSUM (Abbas et al., 2018), HWMA (Abbas, 2018), TEWMA (Letshedi et al., 2021), and the addition of FIR features to monitor the pH value (Abbas et al.,



2019). Among these developments, the MEC control chart has received increasing attention because it simultaneously exploits the memory structure of EWMA and the cumulative detection strength of CUSUM, making it particularly suitable for detecting small and persistent process shifts.

This limitation becomes more evident when MEC charts are applied to real water production data, where short startup phases, operational variability, and subtle parameter shifts are common, increasing the risk of delayed detection and reduced monitoring effectiveness.

Previous studies on MEC control charts primarily focus on theoretical development and simulation-based performance evaluation, often assuming ideal conditions such as normality and long monitoring horizons. In practice, however, water production processes are characterized by early-stage monitoring conditions and subtle parameter fluctuations, which can pose quality risks if not detected in time. Moreover, standard MEC charts without an initial response adjustment tend to exhibit wider control limits at the start of monitoring, thereby reducing their sensitivity to early process shifts.

The integration of FIR and MFIR features addresses this limitation by temporarily tightening control limits during the initial phase, thereby accelerating out-of-control detection. While FIR-enhanced EWMA and

CUSUM charts have been applied in various industrial contexts, including pH monitoring and coefficient-of-variation control, empirical studies combining MEC with FIR/MFIR for residual chlorine monitoring are scarce. This limitation highlights the need for applied research evaluating the practical performance of MEC-FIR/MFIR charts using real water-production data, which is the focus of the present study.

Based on the above discussion, this study aims to investigate the practical behavior of the Mixed EWMA-CUSUM (MEC) control chart integrated with Fast Initial Response (FIR) and Modified Fast Initial Response (MFIR) features for monitoring residual chlorine levels in water production systems. Specifically, this research focuses on addressing the delayed detection of small, early process shifts during the initial monitoring phase, which is common in real water treatment operations. The contributions of this study are threefold: (i) providing an empirical application of MEC-FIR and MEC-MFIR control charts using real water production data, (ii) comparing their detection characteristics under different parameter settings during the startup phase, and (iii) offering practical insights for selecting more sensitive SPC tools to enhance process stability and compliance with drinking water quality standards. By emphasizing applied monitoring behavior rather than

theoretical optimality, this study bridges the gap between methodological development and the real-world implementation of advanced SPC charts.

B. LITERATURE REVIEW

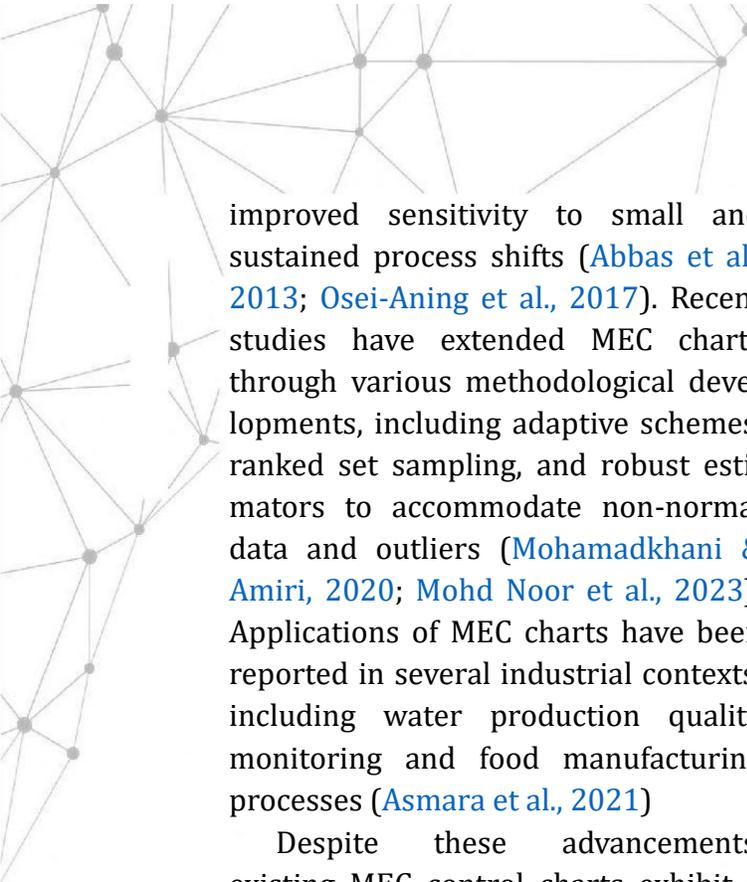
Statistical Process Control (SPC) has been widely adopted as a practical framework for monitoring process stability and detecting abnormal variations in industrial and water production systems. In water treatment operations, SPC plays a crucial role in controlling key water quality parameters, including residual chlorine, which must be maintained within a recommended range to ensure microbiological safety while avoiding adverse health effects (World Health Organization, 2017). Variability in residual chlorine concentration is commonly associated with operational disturbances, changes in raw water quality, and dosing inconsistencies, making continuous monitoring essential for process control (Kwio-Tamale & Onyutha, 2024).

Conventional Shewhart control charts are widely used in practice due to their simplicity; however, numerous studies have reported their limited sensitivity to detecting small, gradual shifts in process parameters, including water quality indicators (Montgomery, 2009). This limitation is particularly critical in residual chlorine monitoring, where subtle deviations may persist for extended periods before triggering an

alarm, potentially compromising water safety. To overcome these shortcomings, more sensitive SPC techniques such as the Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts have been extensively developed and applied.

EWMA charts improve detection performance by assigning greater weight to recent observations, enabling earlier identification of small mean shifts (Lucas & Saccucci, 1990). Similarly, CUSUM charts accumulate deviations from a target value over time, making them effective in detecting persistent changes in the process mean (Page, 1954). Several comparative studies have demonstrated that EWMA and CUSUM charts outperform Shewhart charts in monitoring water quality parameters, including residual chlorine and pH, particularly under small and moderate shift scenarios (de Vargas et al., 2004; Haq et al., 2019). Nevertheless, both EWMA and CUSUM charts may still experience delayed detection during the initial monitoring phase, especially when process shifts occur shortly after chart implementation or system startup.

To further enhance detection capability, hybrid control charts have been proposed by integrating EWMA and CUSUM methodologies. The Mixed EWMA-CUSUM (MEC) control chart combines the memory property of EWMA with the cumulative detection strength of CUSUM, resulting in



improved sensitivity to small and sustained process shifts (Abbas et al., 2013; Osei-Aning et al., 2017). Recent studies have extended MEC charts through various methodological developments, including adaptive schemes, ranked set sampling, and robust estimators to accommodate non-normal data and outliers (Mohamadkhani & Amiri, 2020; Mohd Noor et al., 2023). Applications of MEC charts have been reported in several industrial contexts, including water production quality monitoring and food manufacturing processes (Asmara et al., 2021)

Despite these advancements, existing MEC control charts exhibit a notable limitation during the early stages of monitoring. Several studies indicate that standard MEC charts tend to have wider control limits at the beginning of the monitoring process, thereby reducing their sensitivity to early, small shifts (Abbas et al., 2018; Haq et al., 2014). This limitation becomes more pronounced in real water production systems, where short startup phases and operational variability are common, and delayed detection can increase water quality risks.

Fast Initial Response (FIR) features were introduced to address this issue by temporarily tightening control limits during the initial monitoring phase, thereby accelerating out-of-control detection (Lucas & Crosier, 2000; Steiner, 1999). Modified FIR (MFIR) schemes further enhance this approach by improving detection stability and

reducing false alarms (Haq et al., 2014). FIR and MFIR features have been successfully applied to various control charts, including EWMA, CUSUM, and hybrid schemes, with demonstrated improvements in early detection performance across different industrial applications, such as pH monitoring and coefficient of variation control (Abbas et al., 2019; Hu et al., 2022)

However, empirical studies integrating FIR and MFIR features into MEC control charts for residual chlorine monitoring remain limited. Most existing research focuses on theoretical development or simulation-based performance evaluation under idealized assumptions, with relatively few studies applying these advanced charts to real water-production data. Consequently, there is a lack of empirical evidence regarding the practical behavior of MEC-FIR and MEC-MFIR charts in detecting early-stage shifts in residual chlorine concentration. This study addresses this gap by applying MEC control charts integrated with FIR and MFIR features to real water production data, thereby providing applied insights into their effectiveness for improving early detection and enhancing water quality control.

C. METHODS

1. Exponential Weighted Moving Average Control Chart (EWMA)

The Exponentially Weighted Moving Average (EWMA) control chart was

proposed initially by Roberts in 1959. Analogous to the Cumulative Sum (CUSUM) chart, the EWMA control chart is designed to effectively detect small shifts in the process mean (Montgomery, 2009). The EWMA monitoring statistic is defined as follows:

$$E_i = \lambda x_i + (1 - \lambda)E_{i-1} \quad (1)$$

In equation (1), E_i is the value of EWMA $i=1,2,3,\dots,n$, where n is the number of subgroups. λ_m is the sensitivity parameter or weighting factor of EWMA. The value is in the range of $0 < \lambda_m < 1$. E_0 is the initial number and is set equal to μ_0 (Montgomery, 2009). There are three limits on the EWMA control chart, namely the upper control limit, or also often called the UCL, the middle control limit, which is the same as the target value (μ_0), while the lower control limit is also usually called the LCL. The formula for the EWMA control limits can be seen in the equation below:

$$\begin{aligned} UCL &= L\sigma \sqrt{\frac{\lambda_m}{2-\lambda_m} [1 - (1 - \lambda_m)^{2i}]} + \mu_0 \\ CL &= \mu_0 \\ LCL &= -L\sigma \sqrt{\frac{\lambda_m}{2-\lambda_m} [1 - (1 - \lambda_m)^{2i}]} + \mu_0 \end{aligned} \quad (2)$$

L in equation (2) serves as the diffusion coefficient between the upper and lower for a given percentage of false alarm. The value $L = 3$ is judged to work reasonably well (Montgomery, 2009).

2. The Cumulative Sum Control Chart (CUSUM)

The Cumulative Sum (CUSUM) control chart was introduced initially by Page in 1954. CUSUM control charts are suitable for detecting small, continuous shifts in a process. The CUSUM control chart uses the cumulative deviation from the target value (Montgomery 2009). Tabular CUSUM, which is one of the presentations for the CUSUM control chart, is used by accumulating shifts above the target value, denoted by C_i^+ and C_i^- for shifts below the target value (Montgomery 2009). The value of C_i^+ and C_i^- can be calculated from:

$$\begin{aligned} C_i^+ &= [(x)_i - \mu_0] - M + C_{i-1}^+ \\ C_i^- &= -(x_i - \mu_0) - M + C_{i-1}^- \end{aligned} \quad (3)$$

where C_i^+ : upper of the i -th CUSUM; C_i^- : lower of the i -th CUSUM; x_i : i -th observation; μ_0 : target value and $C_0^+ = C_0^- = 0$: initial value

The preparation of the CUSUM control chart requires two parameters: M and H . The M value is a reference value (slack value) obtained from:

$$M = k\sigma \quad (4)$$

Some previous studies recommend k values of 0.5 or 0.4 (Montgomery 2009). While h is the decision interval or control limit, namely:

$$\begin{aligned} UCL &= h\sigma \\ CL &= 0 \\ LCL &= -h\sigma \end{aligned} \quad (5)$$

Using a value of $h = 4$ or $h = 5$ will generally give the CUSUM control chart

good ARL properties against a shift of about 1σ in the process mean.

3. Mixed EWMA-CUSUM Control Chart (MEC)

The Mixed Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control chart, or the MEC control chart, is constructed by combining the characteristics of the EWMA and CUSUM control chart designs. The MEC value is obtained from MEC_i^+ and MEC_i^- with $i=1,2,3,\dots,n$ from the CUSUM control chart whose initial value is set to zero ($MEC_i^+ = MEC_i^- = 0$) and whose value depends on the value of Q_i from the EWMA control chart in equation (1).

$$Q_i = \lambda_n Y_i + (1 - \lambda_n) Q_{i-1} \quad (6)$$

where λ_n is constant in the range $0 < \lambda_n < 1$ and Q_0 value is equal to the target value ($Q_0 = \mu_0$). Furthermore, the variance of Q_i .

$$Var(Q_i) = \sigma_Y^2 \left(\frac{\lambda_n}{2-\lambda_n} [1 - (1 - \lambda_n)^{2i}] \right) \quad (7)$$

Then the MEC value obtained from the CUSUM feature in equation (3) is defined as follows (Abbas et al., 2013):

$$\begin{aligned} MEC_i^+ &= \max[0, (Q_i - \mu_0) - a_i + MEC_{i-1}^+] \\ MEC_i^- &= \max[0, -(Q_i - \mu_0) - a_i + MEC_{i-1}^-] \end{aligned} \quad (8)$$

Value of a_i can be defined as follows:

$$\begin{aligned} a_i &= a^* \sqrt{Var(Q_i)} = \\ a^* \sigma_Y \sqrt{\frac{\lambda_n}{2-\lambda_n} [1 - (1 - \lambda_n)^{2i}]} \end{aligned} \quad (9)$$

While the control limits on the MEC control chart are defined as follows:

$$\begin{aligned} b_i &= b^* \sqrt{Var(Q_i)} = \\ b^* \sigma_Y \sqrt{\frac{\lambda_n}{2-\lambda_n} [1 - (1 - \lambda_n)^{2i}]} \end{aligned} \quad (10)$$

The b^* and a^* values are the same constants as the h and k values on the CUSUM control chart. Furthermore, the values of MEC_i^+ dan MEC_i^- obtained in equation (8) are plotted against b_i in equation (10), which are the control limits on the mixed EWMA-CUSUM control chart (Abbas et al., 2013).

4. Fast Initial Response (FIR)

The fast initial response (FIR) feature is a feature of EWMA and CUSUM control charts that provides a faster initial response to out-of-control situations than standard EWMA and CUSUM (Lucas & Crosier, 2000). The FIR adjustment introduced is not applied to $C_0^+ = C_0^-$ (in CUSUM) or $Z_0^+ = Z_0^-$ (in EWMA) but the FIR feature is added to the control limits (Steiner, 1999). The form of FIR introduced by Steiner is known as basic fast initial response (BFIR), namely:

$$BFIR_{adj} = [1 - (1 - f)^{(1+a)(i-1)}], 0 \leq f < 1, i = 1, 2, 3, \dots, \text{ dan } a > 0 \quad (11)$$

As a development of BFIR, the modified fast initial response (MFIR) is known and can be calculated as (Haq et al., 2014).

$$MFIR_{adj} = [1 - (1 - f)^{1+a(i-1)}]^{1+\frac{1}{i}}, i = 1, 2, 3, \dots, n \text{ dan } a > 0 \quad (12)$$

for $0 \leq f < 1$, and the value of a can be calculated by:

$$a = -\frac{1}{19} \left(1 + \frac{2}{\log_{10}(1-f)} \right) \quad (13)$$

FIR tuning makes the control limit for the first observation ($i = 1$) the proportion of f at the initial distance from the starting point (Steiner, 1999).

5. Data Collection

The data used in this study were obtained from PDAM Tirta Je'ne'berang, Gowa Regency, Indonesia. The dataset consists of secondary residual chlorine concentration measurements collected during routine water quality monitoring activities in the water production system.

The data collection period spans from 2 January to 11 April 2023, representing the early operational monitoring phase of the water treatment process. This period is particularly relevant for evaluating the performance of control charts with enhanced initial-response features, as process variability and early-stage shifts are more likely to occur during startup and initial stabilization.

A total of 80 observations ($n = 80$) were analyzed in this study. The data were recorded as individual observations, rather than subgrouped measurements. Accordingly, individual-based control chart formulations were

employed, and process variability was estimated using appropriate methods for individual data, consistent with standard SPC practice.

Prior to implementing the control chart, preliminary data screening was conducted. A normality assessment was performed to examine the distributional characteristics of the residual chlorine data and to ensure the absence of extreme deviations that could adversely affect control chart performance. Formal stationarity testing was not conducted, as the primary objective of Statistical Process Control is process monitoring and shift detection rather than time-series modeling or forecasting. In practical SPC applications, particularly for EWMA, CUSUM, and MEC-based control charts, non-stationary behavior often reflects genuine operational changes that should be detected rather than removed through pre-processing.

Using real operational data from a water production system, this study evaluates the practical monitoring behavior of the MEC control chart integrated with FIR and MFIR features under realistic conditions, thereby enhancing the applied relevance of the findings.

D. RESULTS AND DISCUSSION

This section presents the empirical results of applying the MEC control chart with BFIR and MFIR features to residual chlorine data from PDAM Tirta Je'ne'berang Gowa. The results are

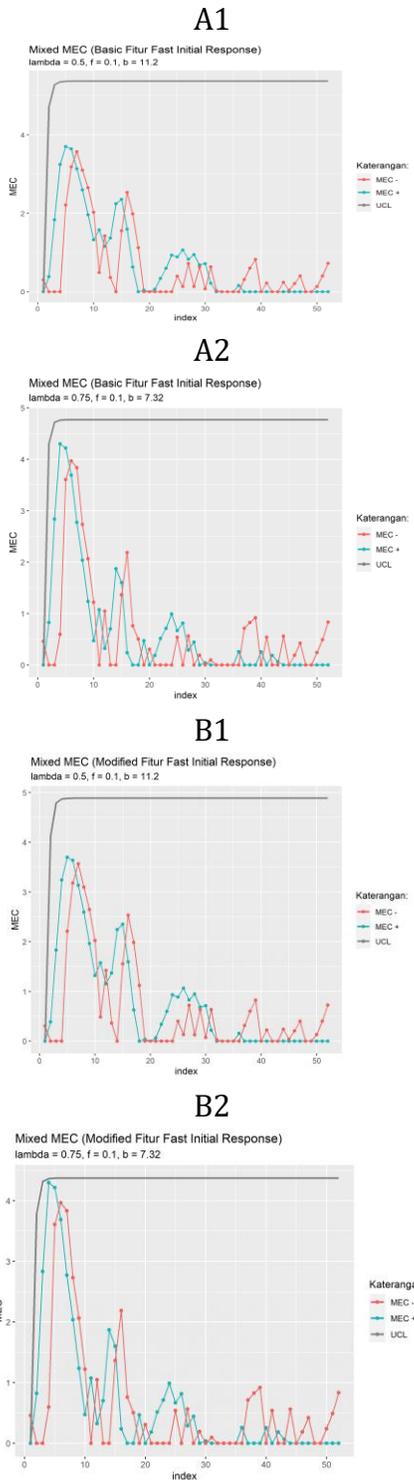


Figure 1. Comparison of MEC control charts with BFIR (A) and MFIR (B) features for $f = 0.1$ and $\lambda_n = 0.5, 0.75$

organized to highlight: (i) detection behavior during the initial monitoring phase, (ii) the comparative performance of BFIR and MFIR features under different parameter settings, and (iii) the practical implications of early detection for water production monitoring.

Figure 1 illustrates the MEC control chart with BFIR and MFIR features for $f = 0.1$ and smoothing parameter values of $\lambda_n = 0.5$ and 0.75 . Panel A presents the MEC control charts with the BFIR feature, where A1 corresponds to $f = 0.1$ and $\lambda_n = 0.5$, and A2 corresponds to $f = 0.1$ and $\lambda_n = 0.75$. In contrast, panel B displays the MEC control charts with the MFIR feature, where B1 represents $f = 0.1$ and $\lambda_n = 0.5$, and B2 represents $f = 0.1$ and $\lambda_n = 0.75$. Although neither feature detects any out-of-control (OOC) signals, the MFIR feature consistently produces narrower control limits compared to the BFIR feature.

Figure 2 presents the MEC control charts incorporating the BFIR and MFIR features for $f = 0.25$ with smoothing parameter values of $\lambda_n = 0.5$ and 0.75 . Panel A displays the MEC control charts with the BFIR feature, where A1 corresponds to $f = 0.25$ and $\lambda_n = 0.5$, and A2 corresponds to $f = 0.25$ and $\lambda_n = 0.75$. In contrast, panel B shows the MEC control charts with the MFIR feature, where B1 represents $f = 0.25$ and $\lambda_n = 0.5$, and B2 represents $f = 0.25$ and $\lambda_n = 0.75$. It is observed that the MFIR feature produces narrower

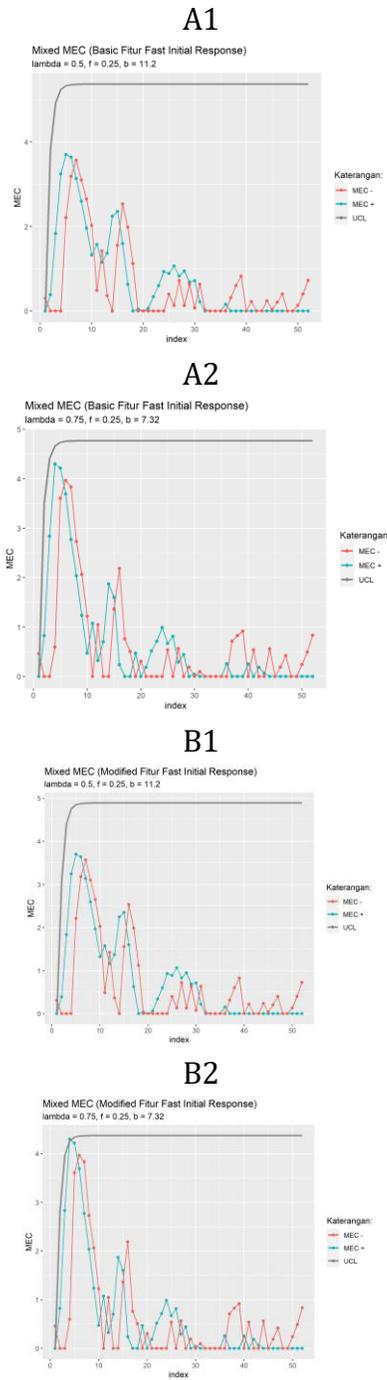


Figure 2. Comparison of MEC control charts with BFIR (A) and MFIR (B) features for $f = 0.25$ and $\lambda_n = 0.5, 0.75$

control limits than the BFIR feature. Moreover, in panel B2, corresponding to

$f = 0.25$ and $\lambda_n = 0.75$, the MFIR-based chart detects one observation exceeding the control limits, indicating the occurrence of an out-of-control (OOC) signal.

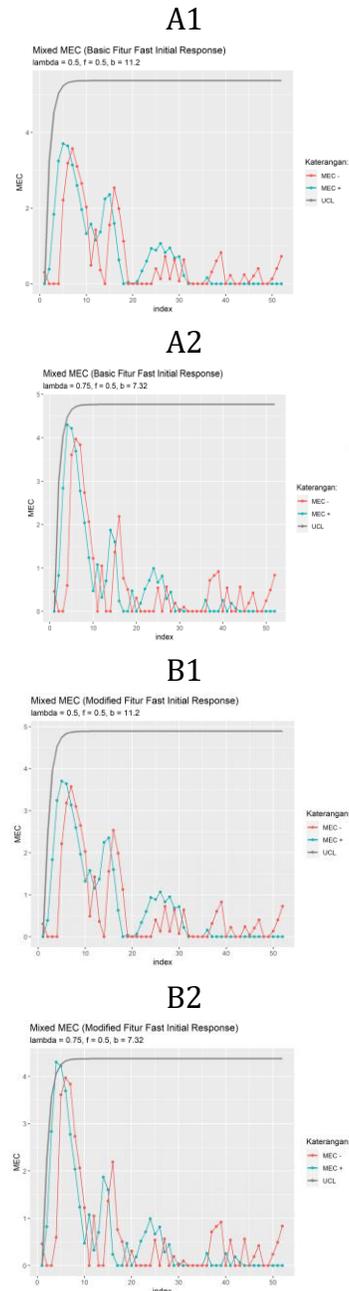


Figure 3. Comparison of MEC control charts with BFIR (A) and MFIR (B) features for $f = 0.5$ and $\lambda_n = 0.5, 0.75$

Figure 3 illustrates the MEC control charts incorporating the BFIR and MFIR features for $f = 0.5$ with smoothing parameter values of $\lambda_n = 0.5$ and 0.75 . Panel A presents the MEC control charts based on the BFIR feature, where A1 corresponds to $f = 0.5$ and $\lambda_n = 0.5$, and A2 corresponds to $f = 0.5$ and $\lambda_n = 0.75$. Panel B shows the MEC control charts using the MFIR feature, with B1 representing $f = 0.5$ and $\lambda_n = 0.5$, and B2 representing $f = 0.5$ and $\lambda_n = 0.75$. The results indicate that the MFIR feature produces narrower control limits compared to the BFIR feature. Furthermore, in panel B2, corresponding to $f = 0.5$ and $\lambda_n = 0.75$, the MFIR-based chart detects two observations exceeding the control limits, indicating the occurrence of out-of-control (OOC) signals.

Figure 4 presents the MEC control charts incorporating the BFIR and MFIR features for $f = 0.75$ with smoothing parameter values of $\lambda_q = 0.5$ and 0.75 . Panel A displays the MEC control charts based on the BFIR feature, where A1 corresponds to $f = 0.75$ and $\lambda_q = 0.5$, and A2 corresponds to $f = 0.75$ and $\lambda_q = 0.75$. Panel B shows the MEC control charts using the MFIR feature, with B1 representing $f = 0.75$ and $\lambda_q = 0.5$, and B2 representing $f = 0.75$ and $\lambda_q = 0.75$. It is evident that the MFIR feature yields narrower control limits than the BFIR feature. Moreover, in panel B2, corresponding to $f = 0.75$ and $\lambda_q = 0.75$, the MFIR-based chart

identifies three observations that exceed the control limits, indicating the occurrence of out-of-control (OOC) signals.

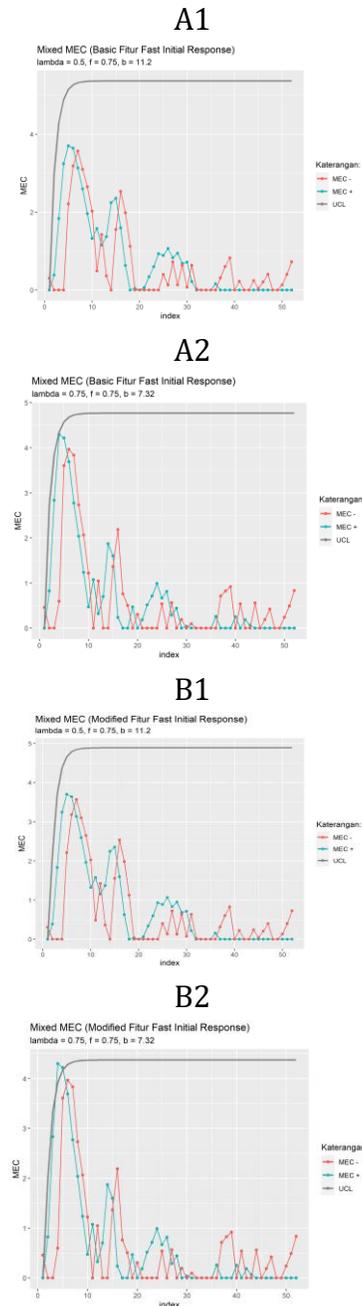


Figure 4. Comparison of MEC control charts with BFIR (A) and MFIR (B) features for $f = 0.75$ and $\lambda_q = 0.5, 0.75$

Table 1. Comparison of BFIR and MFIR features on the MEC control chart

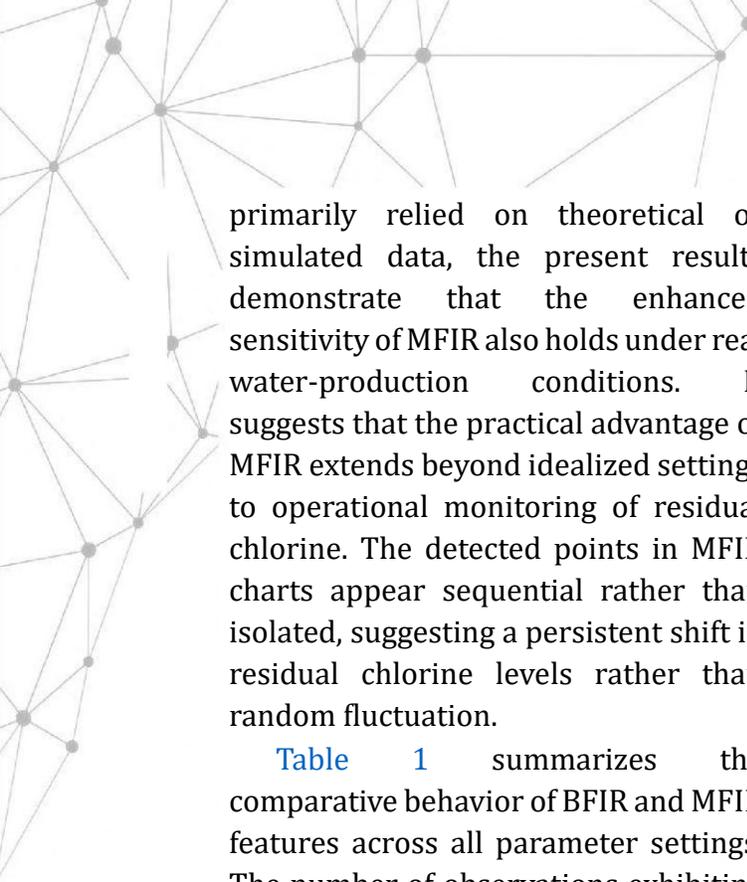
f	λ_q	UCL		Out of control	
		BFIR	MFIR	BFIR	MFIR
0.1	0.1	-	Narrower (7 spots)	-	-
	0.25	-	Narrower (7 spots)	-	-
	0.5	-	Narrower (7 spots)	-	-
	0.75	-	Narrower (7 spots)	-	-
0.25	0.1	-	Narrower (12 spots)	-	-
	0.25	-	Narrower (12 spots)	-	-
	0.5	-	Narrower (12 spots)	-	-
	0.75	-	Narrower (12 spots)	-	1 plotting point
0.5	0.1	-	Narrower (17 spots)	-	-
	0.25	-	Narrower (17 spots)	-	-
	0.5	-	Narrower (17 spots)	-	-
	0.75	-	Narrower (17 spots)	-	2 plotting points
0.75	0.1	-	Narrower (19 spots)	-	-
	0.25	-	Narrower (19 spots)	-	-
	0.5	-	Narrower (19 spots)	-	-
	0.75	-	Narrower (19 spots)	-	3 plotting points

Across all parameter combinations examined, the standard MEC chart integrated with FIR features exhibits distinct detection behavior during the startup phase. For smaller values of the FIR parameter ($f = 0.1$), neither BFIR nor MFIR charts generated out-of-control (OOC) signals, indicating stable monitoring behavior under conservative initial adjustments. However, even in the absence of OOC signals, MFIR consistently produced narrower control limits than BFIR, suggesting greater sensitivity to potential early process changes.

As the FIR parameter increased ($f = 0.25, 0.5, \text{ and } 0.75$), clear differences emerged between BFIR and MFIR features. While BFIR-based MEC charts did not generate any OOC signals across

all examined combinations, MFIR-based charts began to detect process deviations, particularly when paired with higher smoothing parameters ($\lambda_n = 0.75$). Specifically, MFIR detected 1, 2, and 3 OOC observations for $f = 0.25, 0.5, \text{ and } 0.75$, respectively.

These results indicate that MFIR not only narrows the control limits during the startup phase but also translates this increased sensitivity into earlier and more frequent detection of potential process shifts. This empirical finding is consistent with previous simulation-based studies reporting that MFIR produces tighter initial control limits and faster detection than BFIR, particularly during the startup phase (Haq et al., 2014; Steiner, 1999). However, while earlier studies



primarily relied on theoretical or simulated data, the present results demonstrate that the enhanced sensitivity of MFIR also holds under real water-production conditions. It suggests that the practical advantage of MFIR extends beyond idealized settings to operational monitoring of residual chlorine. The detected points in MFIR charts appear sequential rather than isolated, suggesting a persistent shift in residual chlorine levels rather than random fluctuation.

Table 1 summarizes the comparative behavior of BFIR and MFIR features across all parameter settings. The number of observations exhibiting narrower control limits under MFIR increases with higher f values, reaching up to 19 observations for $f = 0.75$. This pattern shows that larger FIR adjustment factors amplify the MEC chart's early-stage sensitivity.

From a practical perspective, the earlier detection achieved by the MEC-MFIR chart is particularly important for residual chlorine monitoring, where delayed response may lead to prolonged exposure to suboptimal disinfectant levels. The results suggest that MFIR-enhanced MEC charts provide a more responsive monitoring tool during the critical startup phase of water production processes.

D. CONCLUSION AND RECOMMENDATION

This study demonstrates that effective monitoring of residual

chlorine in water production requires a statistical quality control method capable of detecting small and early process shifts, particularly during the startup phase. By applying the Mixed EWMA-CUSUM (MEC) control chart integrated with Fast Initial Response (FIR) and Modified Fast Initial Response (MFIR) features to real water production data, this research provides empirical evidence of enhanced early detection capability under practical operating conditions.

From a scientific perspective, the main contribution of this study lies in extending the application of MEC-FIR/MFIR control charts beyond simulation-based settings to real residual chlorine monitoring data. The results confirm that MFIR consistently produces narrower control limits and earlier detection of potential process shifts than BFIR, especially for higher values of the FIR parameter and the smoothing constant. This finding strengthens existing theoretical results by demonstrating that the advantages of MFIR persist under real-world process variability.

From a practical perspective, the results indicate that the MEC-MFIR control chart offers a more responsive monitoring tool for water production operators. Earlier detection of deviations in residual chlorine levels allows faster corrective actions, reducing the risk of prolonged exposure to suboptimal disinfection conditions. Therefore, the proposed

approach can support more reliable operational decision-making and improve compliance with drinking water quality standards.

This study focuses on empirical detection behavior rather than theoretical optimality metrics such as Average Run Length (ARL). While ARL is widely used in SPC design, early identification of potential process shifts during startup is operationally more critical in water production systems. Accordingly, the performance comparison in this study is based on observed detection timing, number of out-of-control signals, and control limit behavior.

For future research, several extensions are recommended. First, incorporating ARL-based analysis would allow a complementary theoretical evaluation of the proposed approach. Second, the method could be extended to multivariate or autocorrelated water quality data, as well as to non-normal or seasonal conditions commonly observed in water treatment processes. Finally, comparative studies involving other advanced SPC methods and longer-term, multi-site datasets would further enhance the generalizability and robustness of the findings.

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